## Recitation 3: Second (Non-homogeneous) Order ODE

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Exercise 1. Find the general solution of the given differential equation.

1. $y^{\prime \prime}-2 y+y=0$;
2. $9 y^{\prime \prime}+6 y^{\prime}+y=0$.

Exercise 2. Consider the initial value problem

$$
y^{\prime \prime}-y^{\prime}+\frac{1}{4}=0, \quad y(0)=2, \quad y^{\prime}(0)=b .
$$

Find the solution as a function of b, and then determine the critical value of b that separates solutions that remain positive for all $t>0$ from those that eventually become negative.

Exercise 3. Use the method of variation of parameters to find a particular solution of the given differential equation

1. $y^{\prime \prime}-5 y^{\prime}+6 y=2 e^{t}$;
2. $y^{\prime \prime}+2 y^{\prime}+y=3 e^{-t}$.

Exercise 4. Use the method of undetermined coefficients to find the general solution of the given differential equation

1. $y^{\prime \prime}+2 y^{\prime}+5 y=3 \sin (2 t)$;
2. $y^{\prime \prime}-2 y^{\prime}-3 y=-3 t e^{-t}$.

Exercise 5. Find the general solution of the given differential equation

$$
y^{\prime \prime}+4 y^{\prime}+4 y=t^{-2} e^{-2 t}, \quad t>0
$$

